Digital Communication Systems
ECS 452

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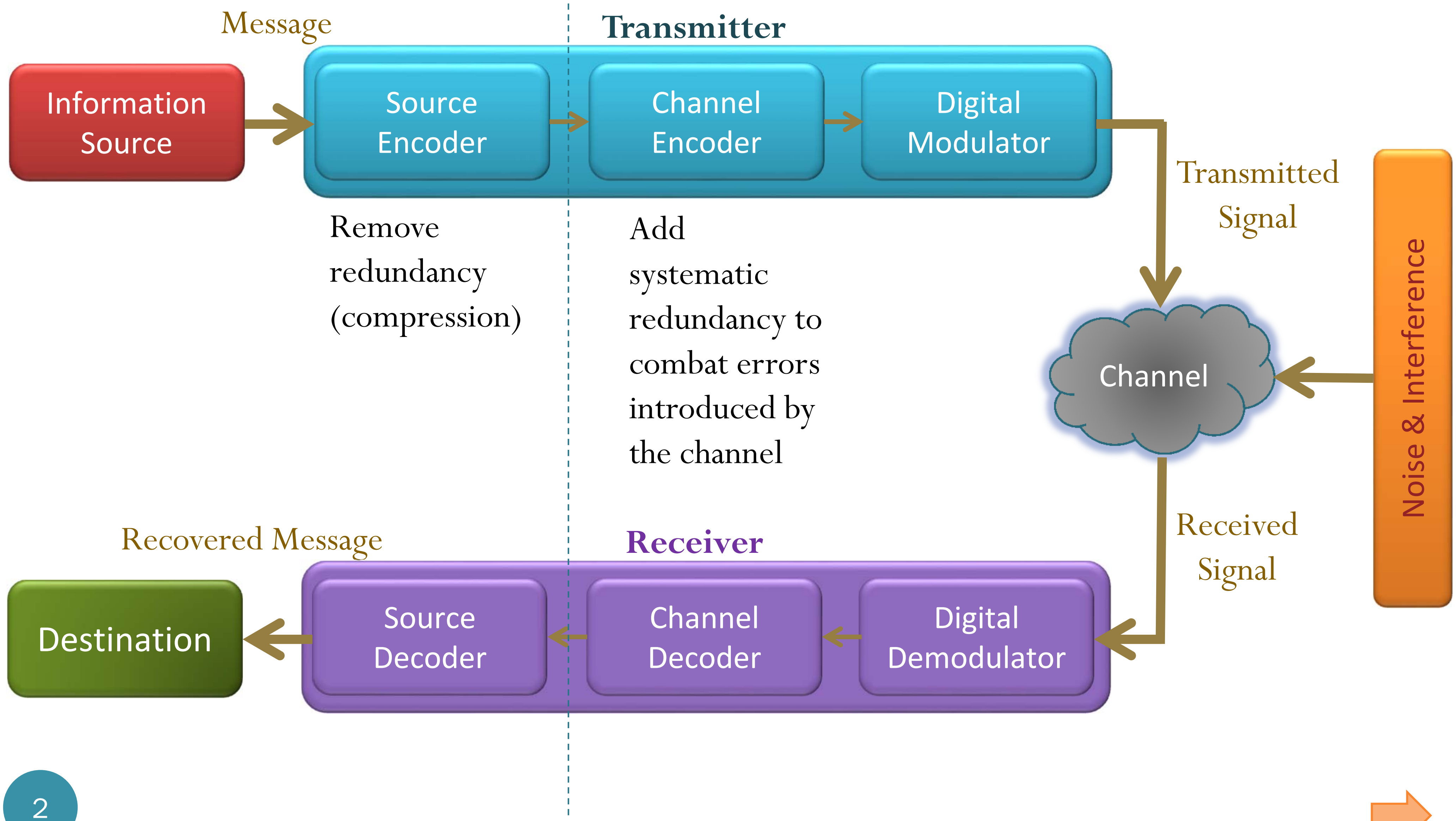
3 Discrete Memoryless Channel DMC)

Office Hours:

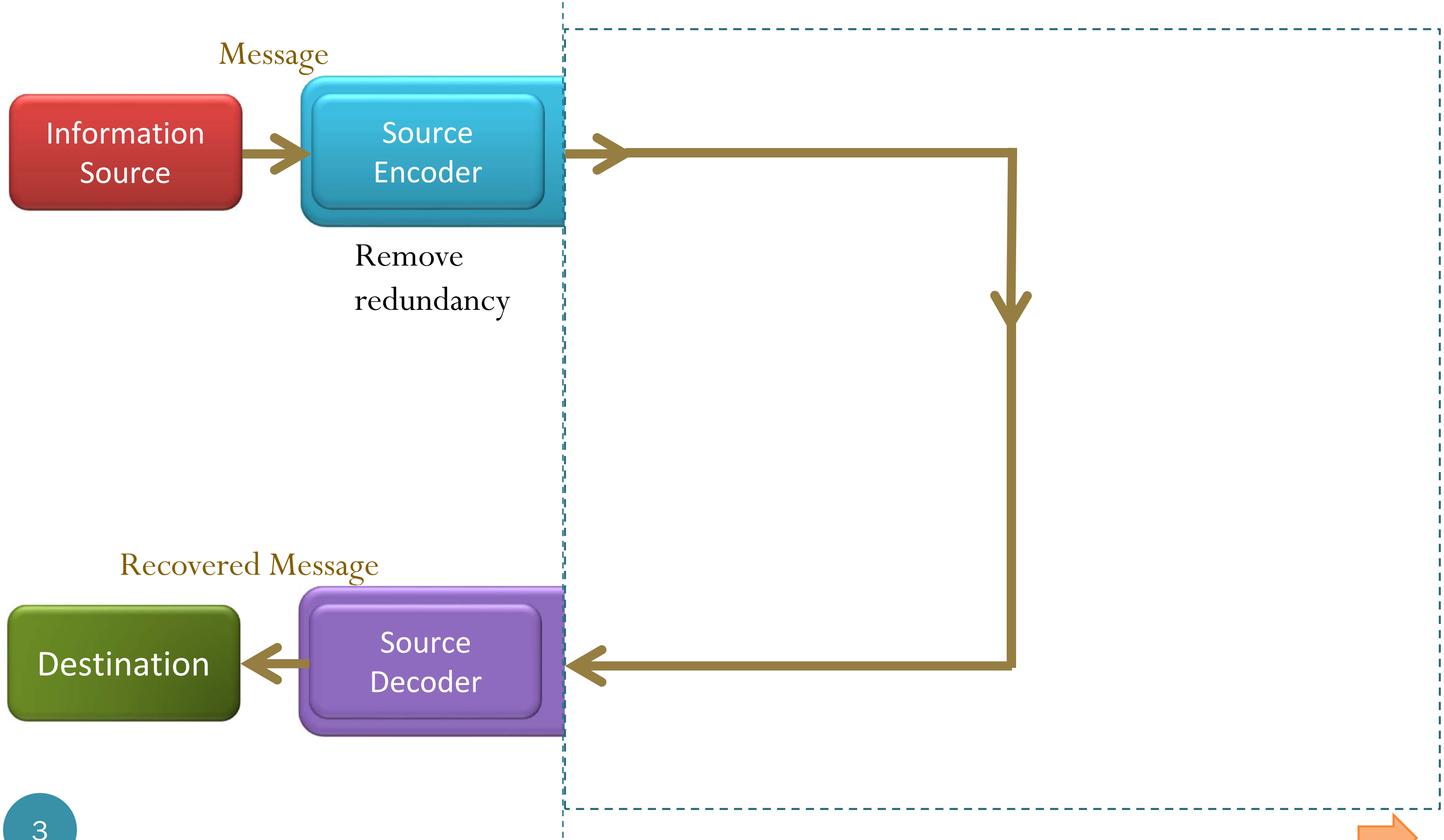
BKD, 4th floor of Sirindhralai building

Monday	14:00-16:00
Thursday	10:30-11:30
Friday	12:00-13:00

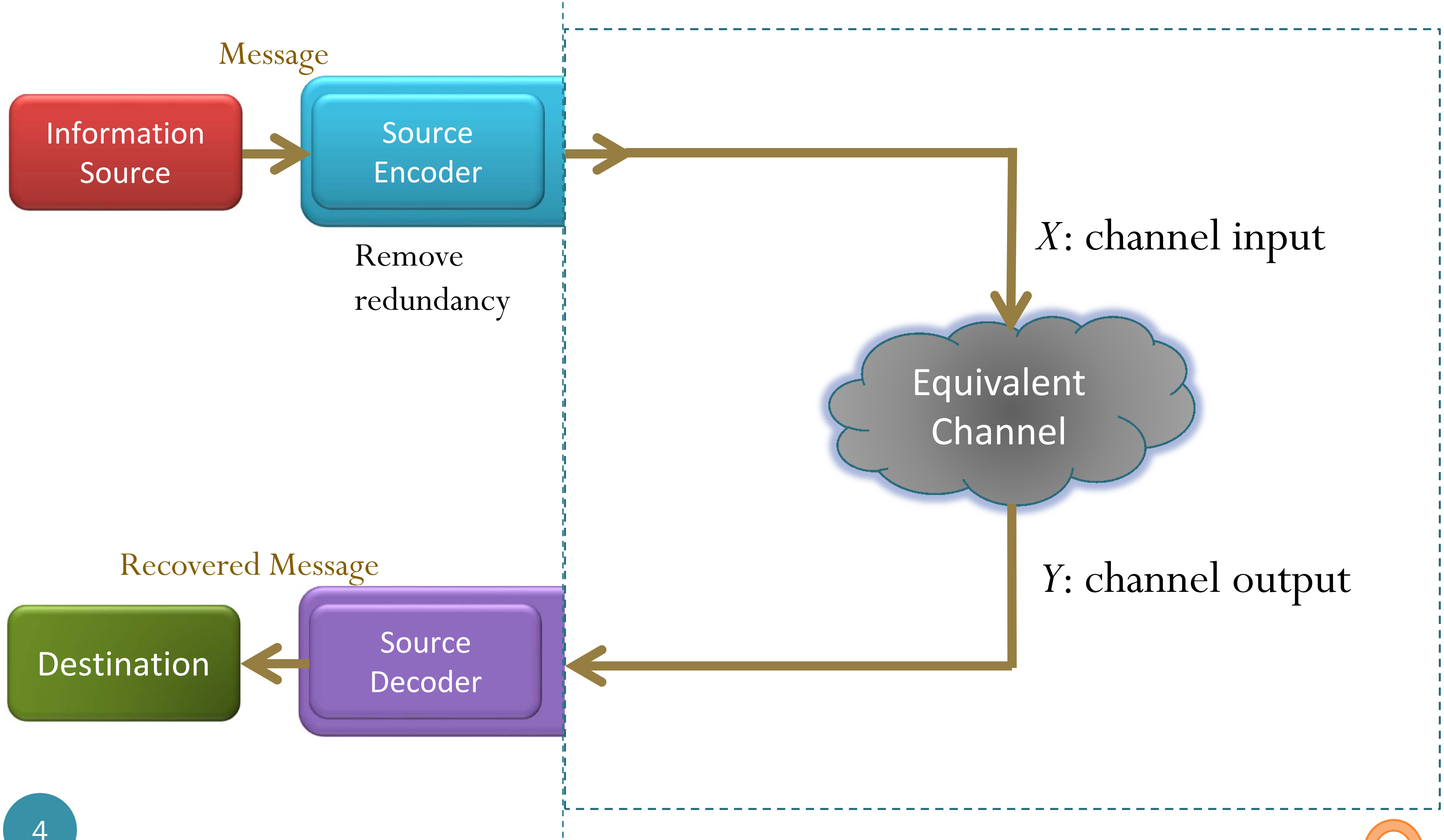
Elements of digital commu. sys.



System considered previously



System considered in this section



MATLAB

```
%% Generating the channel input x
x = randsrc(1,n,[S_X;p_X]); % channel input

%% Applying the effect of the channel to create the channel output y
y = DMC_Channel_sim(x,S_X,S_Y,Q); % channel output
```

```
function y = DMC_Channel_sim(x,S_X,S_Y,Q)
%% Applying the effect of the channel to create the channel output y
y = zeros(size(x)); % preallocation
for k = 1:length(x)
    % Look at the channel input one by one. Choose the corresponding row
    % from the Q matrix to generate the channel output.
    y(k) = randsrc(1,1,[S_Y;Q(find(S_X == x(k)),:)]);
end
```

[DMC_Channel_sim.m]

[DMC_Analysis_demo.m]

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 20;
% Channel Input
S_X = [0 1]; S_Y = [0 1];
p_X = [0.3 0.7];
% Channel Characteristics
p = 0.1; Q = [1-p p; p 1-p];
```

[Example 3.2]

Ex: BSC

```
>> BSC_demo
ans =
1 0 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1
ans =
1 1 1 1 1 1 1 1 1 0 1 1 0 1 0 1 1 1 1 1

p_X =
    0.3000    0.7000

Q =
    0.9000    0.1000
    0.1000    0.9000

q =
    0.3400    0.6600

Elapsed time is 0.134992 seconds.
```

[BSC_demo.m]

Rel. freq. from the simulation

```
%% Statistical Analysis
% The probability values for the channel inputs
p_X                        % Theoretical probability
p_X_sim = hist(x,S_X)/n % Relative frequencies from the simulation
% The probability values for the channel outputs
q = p_X*Q                  % Theoretical probability
q_sim = hist(y,S_Y)/n   % Relative frequencies from the simulation
% The channel transition probabilities from the simulation
Q_sim = [];
for k = 1:length(S_X)
    I = find(x==S_X(k)); LI = length(I);
    rel_freq_Xk = LI/n;
    yc = y(I);
    cond_rel_freq = hist(yc,S_Y)/LI; Q_sim = [Q_sim; cond_rel_freq];
end
Q                          % Theoretical probability
Q_sim                      % Relative frequencies from the simulation
```


[DMC_Analysis_demo.m]

[Example 3.2]

Ex: BSC

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 20;
% Channel Input
S_X = [0 1]; S_Y = [0 1];
p_X = [0.3 0.7];
% Channel Characteristics
p = 0.1; Q = [1-p p; p 1-p];
```

```
>> BSC_demo
ans =
1 0 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1
ans =
1 1 1 1 1 1 1 1 1 0 1 1 0 1 0 1 1 1 1 1

p_X =
    0.3000    0.7000
p_X_sim =
    0.1500    0.8500
q =
    0.3400    0.6600
q_sim =
    0.1500    0.8500

Q =
    0.9000    0.1000
    0.1000    0.9000
Q_sim =
    0.6667    0.3333
    0.0588    0.9412
```

Because there are only 20 samples, we can't expect the relative freq. from the simulation to match the specified or calculated probabilities.

```
Elapsed time is 0.134992 seconds.
```

[BSC_demo.m]

Ex: BSC

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 1e4;
% Channel Input
S_X = [0 1]; S_Y = [0 1];
p_X = [0.3 0.7];
% Channel Characteristics
p = 0.1; Q = [1-p p; p 1-p];
```

```
>> BSC_demo

p_X =
    0.3000    0.7000
p_X_sim =
    0.3037    0.6963
q =
    0.3400    0.6600
q_sim =
    0.3407    0.6593

Q =
    0.9000    0.1000
    0.1000    0.9000
Q_sim =
    0.9078    0.0922
    0.0934    0.9066

Elapsed time is 0.922728 seconds.
```

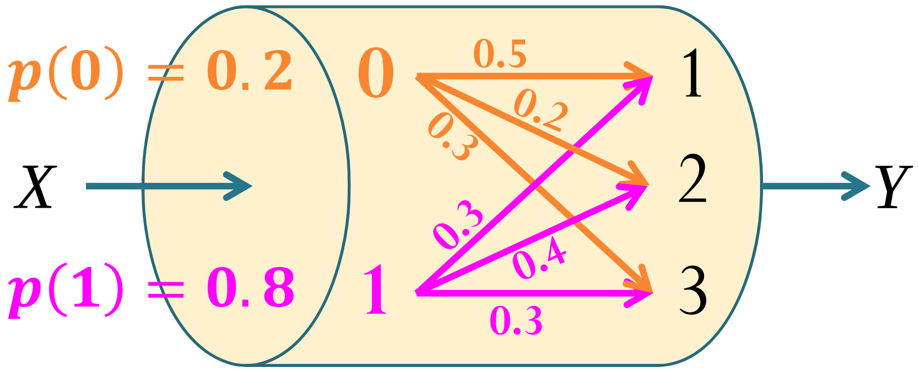

[BSC_demo.m]

Ex: DMC

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 20;
% General DMC
% Ex. 3.16 in lecture note
% Channel Input
S_X = [0 1]; S_Y = [1 2 3];
p_X = [0.2 0.8];
% Channel Characteristics
Q = [0.5 0.2 0.3; 0.3 0.4 0.3];
```

```
>> DMC_demo
ans =
x:  1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 0 1
ans =
y:  1 3 2 2 1 2 1 2 2 3 1 1 1 3 1 3 2 3 1 2
```

```
p_X =
    0.2000    0.8000
p_X_sim =
    0.2000    0.8000
q =
    0.3400    0.3600    0.3000
q_sim =
    0.4000    0.3500    0.2500
Q =
    0.5000    0.2000    0.3000
    0.3000    0.4000    0.3000
Q_sim =
    0.7500         0    0.2500
    0.3125    0.4375    0.2500
```



[DMC_demo.m]

Ex: DMC

```
>> p = [0.2 0.8]
p =
    0.2000    0.8000
>> p = [0.2 0.8];
>> Q = [0.75 0 0.25; 0.3125 0.4375 0.25];
>> p*Q
ans =
    0.4000    0.3500    0.2500
```

Block Matrix Multiplications

$$\left(\underbrace{\begin{matrix}10 & 6\\ 9 & 7\end{matrix}}_{A}\ \underbrace{\begin{matrix}6 & 4 & 3\\ 3 & 5 & 9\end{matrix}}_{B}\right)\times\begin{pmatrix}\underbrace{\begin{matrix}2 & 2 & 5\\ 3 & 3 & 4\end{matrix}}_{C} & \underbrace{\begin{matrix}10 & 2 & 10 & 2 & 5\\ 5 & 10 & 5 & 3 & 6\end{matrix}}_{D}\\ \underbrace{\begin{matrix}3 & 3 & 4\\ 7 & 2 & 5\\ 8 & 3 & 6\end{matrix}}_{E} & \underbrace{\begin{matrix}1 & 1 & 5 & 5 & 6\\ 3 & 10 & 6 & 10 & 3\\ 9 & 8 & 3 & 6 & 5\end{matrix}}_{F}\end{pmatrix}$$

$$=\left(\underbrace{\begin{matrix}108 & 73 & 136\\ 155 & 85 & 164\end{matrix}}_{AC+BE}\ \underbrace{\begin{matrix}175 & 150 & 193 & 126 & 149\\ 224 & 213 & 197 & 158 & 165\end{matrix}}_{AD+BF}\right)$$

$$\underbrace{\begin{pmatrix}10 & 6 & 6 & 4 & 3\\ 9 & 7 & 3 & 5 & 9\end{pmatrix}}_{X}\times\left(\underbrace{\begin{matrix}2 & 2 & 5 & 10\\ 3 & 3 & 4 & 5\\ 3 & 3 & 4 & 1\\ 7 & 2 & 5 & 3\\ 8 & 3 & 6 & 9\end{matrix}}_{G}\ \underbrace{\begin{matrix}2 & 10 & 2 & 5\\ 10 & 5 & 3 & 6\\ 1 & 5 & 5 & 6\\ 10 & 6 & 10 & 3\\ 8 & 3 & 6 & 5\end{matrix}}_{H}\right)$$

$$=\left(\underbrace{\begin{matrix}108 & 73 & 136 & 175\\ 155 & 85 & 164 & 224\end{matrix}}_{XG}\ \underbrace{\begin{matrix}150 & 193 & 126 & 149\\ 213 & 197 & 158 & 165\end{matrix}}_{XH}\right)$$

Review: Evaluation of Probability from the Joint PMF Matrix

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$$P_{X,Y} \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0.1 & 0.1 & 0 & 0 & 0 \\ 3 & 0.1 & 0 & 0 & 0.1 & 0 \\ 4 & 0 & 0.1 & 0.2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0.3 \end{array}$$

- Find $P[X + Y < 7]$

Step 1: Find the pairs (x,y) that satisfy the condition "x+y < 7"

One way to do this is to first construct the matrix of x+y.

$$x + y \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 8 & 9 & 10 \\ 6 & 8 & 9 & 10 & 11 & 12 \end{array}$$

Review: Evaluation of Probability from the Joint PMF Matrix

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$$P_{X,Y} \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0.1 & 0.1 & 0 & 0 & 0 \\ 3 & 0.1 & 0 & 0 & 0.1 & 0 \\ 4 & 0 & 0.1 & 0.2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0.3 \end{array}$$

- Find $P[X + Y < 7]$

Step 2: Add the corresponding probabilities from the joint pmf (matrix)

$$x + y \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 8 & 9 & 10 \\ 6 & 8 & 9 & 10 & 11 & 12 \end{array}$$

$$P[X + Y < 7] = 0.1 + 0.1 + 0.1 = 0.3$$

Review: Evaluation of Probability from the Joint PMF Matrix

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$$P_{X,Y} \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline \begin{matrix} 1 \\ 3 \\ 4 \\ 6 \end{matrix} & \multicolumn{5}{c}{\begin{bmatrix} 0.1 & 0.1 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.1 & 0 \\ 0 & 0.1 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}} \end{array}$$

- Find $P[X = Y]$

$$P[X = Y] = 0 + 0.2 + 0.3 \; = 0.5$$

Review: Sum of two discrete RVs

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$$P_{X,Y} \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0.1 & 0.1 & 0 & 0 & 0 \\ 3 & 0.1 & 0 & 0 & 0.1 & 0 \\ 4 & 0 & 0.1 & 0.2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0.3 \end{array}$$

- Find $P[X+Y=7]$

$P[X+Y=7] = 0.1$

$$x+y \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 8 & 9 & 10 \\ 6 & 8 & 9 & 10 & 11 & 12 \end{array}$$

Ex: DMC

```
>> p = [0.2 0.8];
>> Q = [0.5 0.2 0.3; 0.3 0.4 0.3];
>> p*Q
ans =
    0.3400    0.3600    0.3000
>> P = (diag(p))*Q
P =
    0.1000    0.0400    0.0600
    0.2400    0.3200    0.2400
>> sum(P)
ans =
    0.3400    0.3600    0.3000
```

MATLAB

```
%% Naive Decoder
x_hat = y;
```

```
%% Error Probability
PE_sim = 1-sum(x==x_hat)/n % Error probability from the simulation
```

```
% Calculation of the theoretical error probability
PC = 0;
for k = 1:length(S_X)
    t = S_X(k);
    i = find(S_Y == t);
    if length(i) == 1
        PC = PC+ p_X(k)*Q(k,i);
    end
end
PE_theretical = 1-PC
```

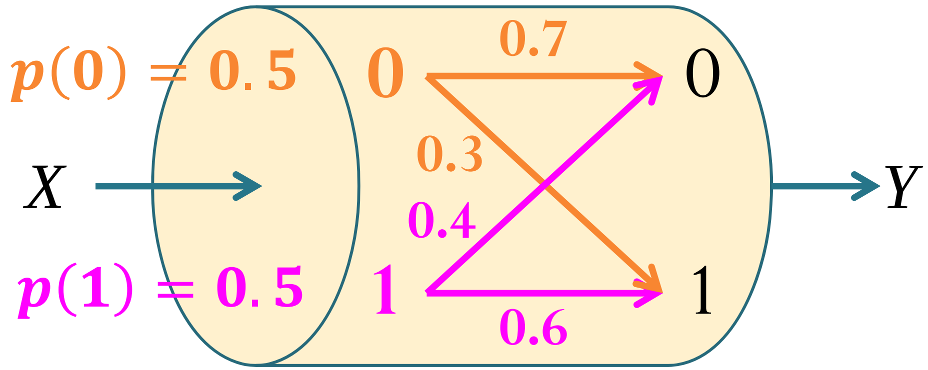

[DMC_Analysis_demo.m]

[Ex. 3.18]

Ex: BAC

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 20;
% Binary Assymmetric Channel (BAC)
% Ex 3.8 in lecture note (11.3 in [Z&T, 2010])
% Channel Input
S_X = [0 1]; S_Y = [0 1];
p_X = [0.5 0.5];
% Channel Characteristics
Q = [0.7 0.3; 0.4 0.6];
```

```
>> BAC_demo
ans =
x:  0 0 0 1 1 0 0 1 0 0 0 0 1 0 0 1 0 1 0 0
ans =
y:  0 0 1 1 0 0 0 1 1 1 0 0 1 0 0 0 0 0 1 0
```




```
p_X =
    0.5000    0.5000
p_X_sim =
    0.7000    0.3000
q =
    0.5500    0.4500
q_sim =
    0.6500    0.3500
Q =
    0.7000    0.3000
    0.4000    0.6000
Q_sim =
    0.7143    0.2857
    0.5000    0.5000
```

$\frac{7}{20}$

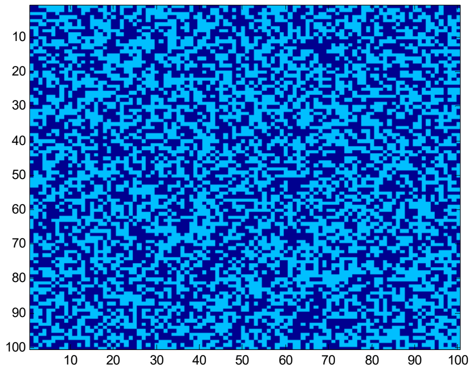
PE_sim =
0.3500
PE_theretical =
0.3500

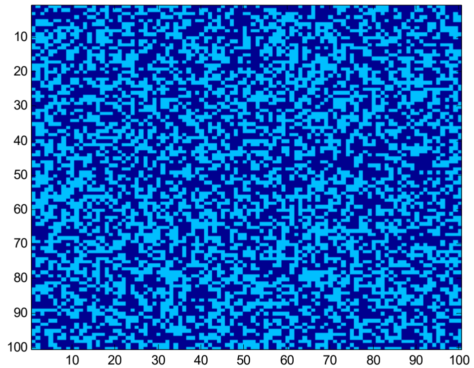
[BAC_demo.m]

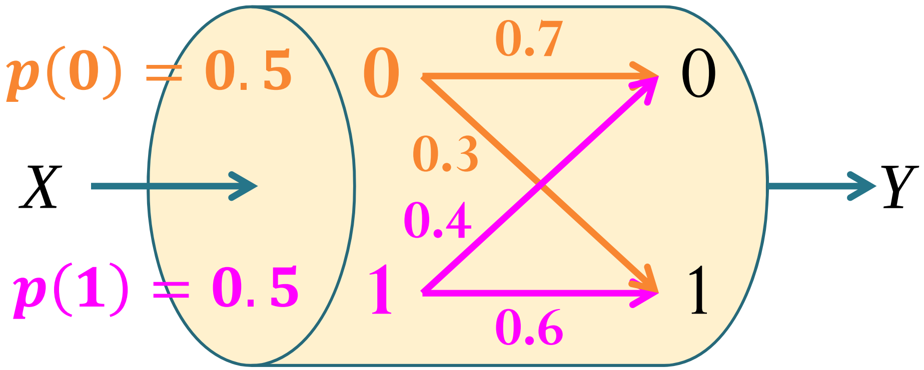
[Ex. 3.18]

Ex: BAC

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 1e4;
% Binary Assymmetric Channel (BAC)
% Ex 3.8 in lecture note (11.3 in [Z&T, 2010])
% Channel Input
S_X = [0 1]; S_Y = [0 1];
p_X = [0.5 0.5];
% Channel Characteristics
Q = [0.7 0.3; 0.4 0.6];
```








```
p_X =
    0.5000    0.5000
p_X_sim =
    0.5043    0.4957
q =
    0.5500    0.4500
q_sim =
    0.5532    0.4468
Q =
    0.7000    0.3000
    0.4000    0.6000
Q_sim =
    0.7109    0.2891
    0.3928    0.6072
PE_sim =
    0.3405
PE_theretical =
    0.3500
```

[BAC_demo.m]

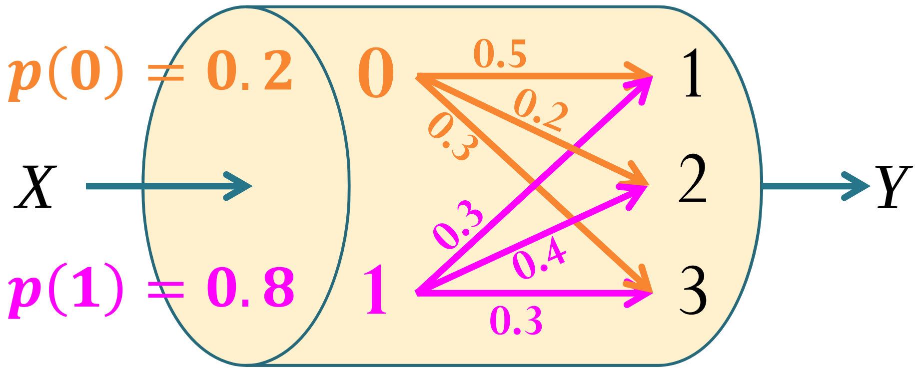
[Ex. 3.21]

Ex: DMC

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 20;
% General DMC
% Ex. 3.16 in lecture note
% Channel Input
S_X = [0 1]; S_Y = [1 2 3];
p_X = [0.2 0.8];
% Channel Characteristics
Q = [0.5 0.2 0.3; 0.3 0.4 0.3];
```

```
>> DMC_demo
ans =
x:  1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 0 1
ans =
y:  1 3 2 2 1 2 1 2 2 3 1 1 1 3 1 3 2 3 1 2
```

[Same samples as in Ex. 3.6]



$$\frac{20-4}{20}$$

```
p_X =
   0.2000   0.8000
p_X_sim =
   0.2000   0.8000
q =
   0.3400   0.3600   0.3000
q_sim =
   0.4000   0.3500   0.2500
Q =
   0.5000   0.2000   0.3000
   0.3000   0.4000   0.3000
Q_sim =
   0.7500        0   0.2500
   0.3125   0.4375   0.2500
```

PE_sim =
0.7500

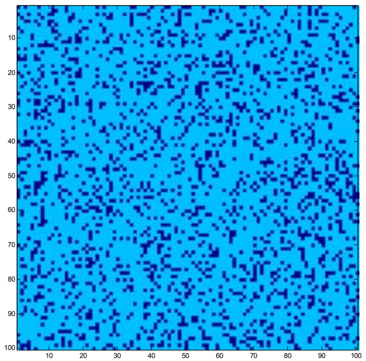
PE_theretical =
0.7600

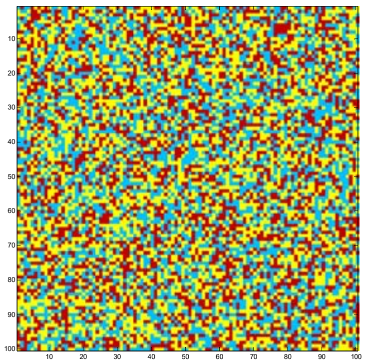
[DMC_demo.m]

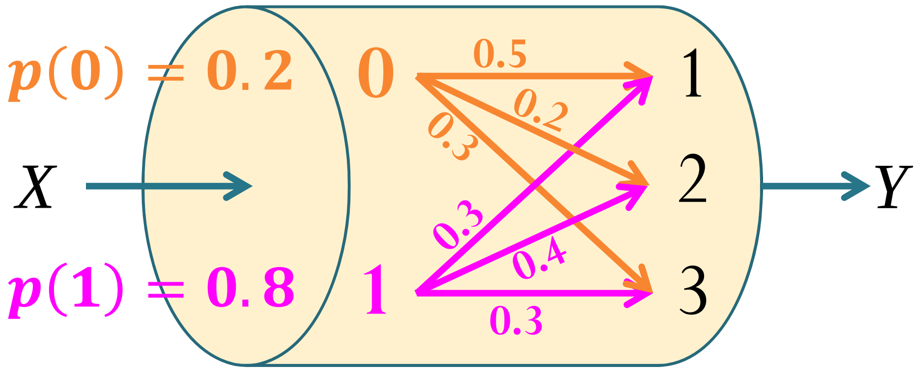
[Ex. 3.21]

Ex: DMC

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 1e4;
% General DMC
% Ex. 3.16 in lecture note
% Channel Input
S_X = [0 1]; S_Y = [1 2 3];
p_X = [0.2 0.8];
% Channel Characteristics
Q = [0.5 0.2 0.3; 0.3 0.4 0.3];
```








```
p_X =
   0.2000   0.8000
p_X_sim =
   0.2011   0.7989
q =
   0.3400   0.3600   0.3000
q_sim =
   0.3387   0.3607   0.3006
Q =
   0.5000   0.2000   0.3000
   0.3000   0.4000   0.3000
Q_sim =
   0.4943   0.1914   0.3143
   0.2995   0.4033   0.2972
```

PE_sim =
0.7607
PE_theretical =
0.7600

[DMC_demo.m]

Optimal Decoder for BSC

```
>> BSC_decoder_ALL_demo
Decoder_Table_ALL =
     0     1
     1     0
     1     1
     0     0
```

$\hat{x}(0)$ $\hat{x}(1)$ $P(\mathcal{E})$

```
ans =
         0    1.0000    0.1000
    1.0000         0    0.9000
    1.0000    1.0000    0.8000
         0         0    0.2000
Optimal_Detector =
     0     1
Min_PE =
    0.1000
Elapsed time is 0.008709 seconds.
```

```
close all; clear all;
tic

%% Simulation parameters
% Channel Input
S_X = [0 1]; S_Y = [0 1];
p0 = 0.8; p1 = 1-p0; p_X = [p0 p1];
% Channel Characteristics
p = 0.1; Q = [1-p p; p 1-p];

%% All possible "reasonable" decoders
% X_hat = Y; X_hat = 1-Y; X_hat = 1; X_hat = 0
Decoder_Table_ALL = [0 1; 1 0; 1 1; 0 0]

%% Calculate the error probability for each of the decoder
PE_ALL = [];
for k = 1:size(Decoder_Table_ALL,1)
    Decoder_Table = Decoder_Table_ALL(k,:);
    PC = 0;
    for k = 1:length(S_X)
        I = (Decoder_Table == S_X(k));
        Q_row = Q(k,:);
        PC = PC + p_X(k)*sum(Q_row(I));
    end
    PE_theretical = 1-PC;
    PE_ALL = [PE_ALL; PE_theretical];
end

%% Display the results
[Decoder_Table_ALL PE_ALL]

%% Find the optimal detectors
[V I] = min(PE_ALL);
Optimal_Detector = Decoder_Table_ALL(I,:)
Min_PE = V

toc
```


[BSC_decoder_ALL_demo.m]

DIY Decoder

```
>> DMC_decoder_DIY_demo
ans =
```

X 1 0 1 1 1 1 1 0 1 1 0 1 1 1 1 0 0 1 0 1

ans =

Y 2 1 1 3 3 1 2 2 1 2 1 2 3 1 1 3 1 3 1 1

ans =

$\hat{X}$ 1 0 0 0 0 0 1 1 0 1 0 1 0 0 0 0 0 0 0 0

PE_sim =

0.5500

PE_theretical =

0.5200

Elapsed time is 0.081161 seconds.

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 20;
% General DMC
% Ex. 3.16 in lecture note
% Channel Input
S_X = [0 1]; S_Y = [1 2 3];
p_X = [0.2 0.8];
% Channel Characteristics
Q = [0.5 0.2 0.3; 0.3 0.4 0.3];
```

```
%% DIY Decoder
Decoder_Table = [0 1 0]; % The decoded
values corresponding to the received Y
```

[DMC_decoder_DIY_demo.m]

DIY Decoder

```
%% DIY Decoder
Decoder_Table = [0 1 0]; % The decoded values corresponding to the received Y
```

```
% Decode according to the decoder table
x_hat = y; % preallocation
for k = 1:length(S_Y)
    I = (y==S_Y(k));
    x_hat(I) = Decoder_Table(k);
end

PE_sim = 1-sum(x==x_hat)/n % Error probability from the simulation
```

```
% Calculation of the theoretical error probability
PC = 0;
for k = 1:length(S_X)
    I = (Decoder_Table == S_X(k));
    q = Q(k,:);
    PC = PC+ p_X(k)*sum(q(I));
end
PE_theretical = 1-PC
```

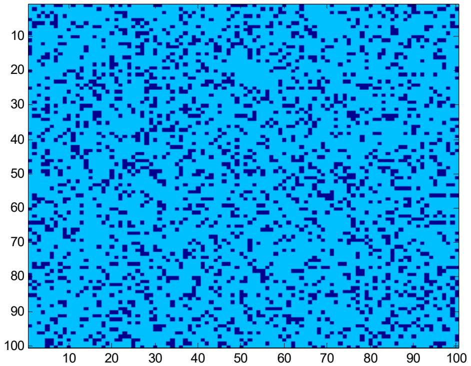
[DMC_decoder_DIY_demo.m]

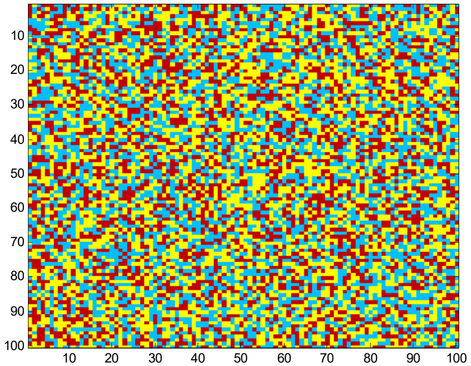
DIY Decoder

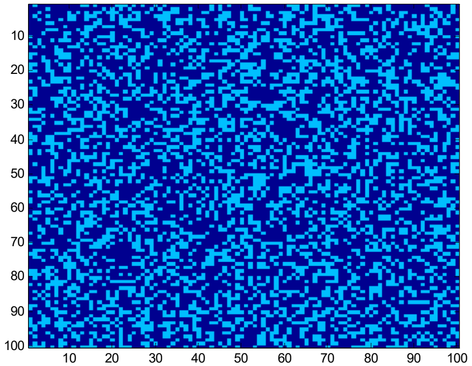
```
>> DMC_decoder_DIY_demo
PE_sim =
   0.5213
PE_theretical =
   0.5200
Elapsed time is 2.154024 seconds.
```

```
%% Simulation parameters
% The number of symbols to be transmitted
n = 1e4;
% General DMC
% Ex. 3.16 in lecture note
% Channel Input
S_X = [0 1]; S_Y = [1 2 3];
p_X = [0.2 0.8];
% Channel Characteristics
Q = [0.5 0.2 0.3; 0.3 0.4 0.3];
```

```
%% DIY Decoder
Decoder_Table = [0 1 0]; % The decoded values corresponding to the received Y
```







[DMC_decoder_DIY_demo.m]

Searching for the Optimal Detector

```
>> DMC_decoder_ALL_demo
ans =    x̂(1)        x̂(2)        x̂(3)        P(ℰ)
              0           0           0      0.8000
              0           0      1.0000      0.6200
              0      1.0000           0      0.5200
              0      1.0000      1.0000      0.3400
         1.0000           0           0      0.6600
         1.0000           0      1.0000      0.4800
         1.0000      1.0000           0      0.3800
         1.0000      1.0000      1.0000      0.2000
Min_PE =
         0.2000
Optimal_Detector =
           1     1     1
Elapsed time is 0.003351 seconds.
```

Review: ECS315 (2015)

6.4. ***Interpretation***: It is sometimes useful to interpret $P(A)$ as our knowledge of the occurrence of event A *before* the experiment takes place. Conditional probability[24] $P(A|B)$ is the ***updated probability*** of the event A given that we now know that B occurred (but we still do not know which particular outcome in the set B did occur).

Definition 6.5. Sometimes, we refer to $P(A)$ as

- **a priori probability**, or
- the **prior probability** of A, or
- the **unconditional probability** of A.

In which case, we refer to $P(A|B)$ as

- a posteriori probability
- posterior probability
- conditional probability

Review: ECS315 (2014)

6.4. ***Interpretation***: Sometimes, we refer to $P(A)$ as

- **a priori probability** , or
- the **prior probability** of A, or
- the **unconditional probability** of A.

$P(A|B)$

a posteriori probability

the posterior probability

conditional probability

It is sometimes useful to interpret $P(A)$ as our knowledge of the occurrence of event A *before* the experiment takes place. Conditional probability $P(A|B)$ is the ***updated probability*** of the event A given that we now know that B occurred (but we still do not know which particular outcome in the set B occurred).

MAP Decoder

```
%% MAP Decoder
P = diag(p_X)*Q; % Weight the channel transition probability by the
                 % corresponding prior probability.
[V I] = max(P); % For I, the default MATLAB behavior is that when there are
                % multiple max, the index of the first one is returned.
Decoder_Table = S_X(I) % The decoded values corresponding to the received Y
```

```
%% Decode according to the decoder table
x_hat = y; % preallocation
for k = 1:length(S_Y)
    I = (y==S_Y(k));
    x_hat(I) = Decoder_Table(k);
end

PE_sim = 1-sum(x==x_hat)/n % Error probability from the simulation
```

```
%% Calculation of the theoretical error probability
PC = 0;
for k = 1:length(S_X)
    I = (Decoder_Table == S_X(k));
    Q_row = Q(k,:);
    PC = PC+ p_X(k)*sum(Q_row(I));
end
PE_theretical = 1-PC
```


[DMC_decoder_MAP_demo.m]

ML Decoder

```
%% ML Decoder
[V I] = max(Q); % For I, the default MATLAB behavior is that when there are
                % multiple max, the index of the first one is returned.
Decoder_Table = S_X(I) % The decoded values corresponding to the received Y
```

```
%% Decode according to the decoder table
x_hat = y; % preallocation
for k = 1:length(S_Y)
    I = (y==S_Y(k));
    x_hat(I) = Decoder_Table(k);
end

PE_sim = 1-sum(x==x_hat)/n % Error probability from the simulation
```

```
%% Calculation of the theoretical error probability
PC = 0;
for k = 1:length(S_X)
    I = (Decoder_Table == S_X(k));
    Q_row = Q(k,:);
    PC = PC+ p_X(k)*sum(Q_row(I));
end
PE_theretical = 1-PC
```


[DMC_decoder_ML_demo.m]

Guessing Game 1

- There are 15 cards.
  - Each have a number on it.
  - Here are the 15 cards:

1	2	2	3	3	3	4	4	4	4	5	5	5	5	5

- One card is randomly selected from the 15 cards.
- You need to guess the number on the card.
- Have to pay 1 Baht for incorrect guess.

- The game is to be repeated $n = 10{,}000$ times.
- What should be your guess value?

```
close all; clear all;

n = 10; % number of time to play this game

D = [1 2 2 3 3 3 4 4 4 4 5 5 5 5 5];
X = D(randi(length(D),1,n));

if n <= 10
    X
end

g = 1
cost = sum(X ~= g)

if n > 1
averageCostPerGame = cost/n
end
```

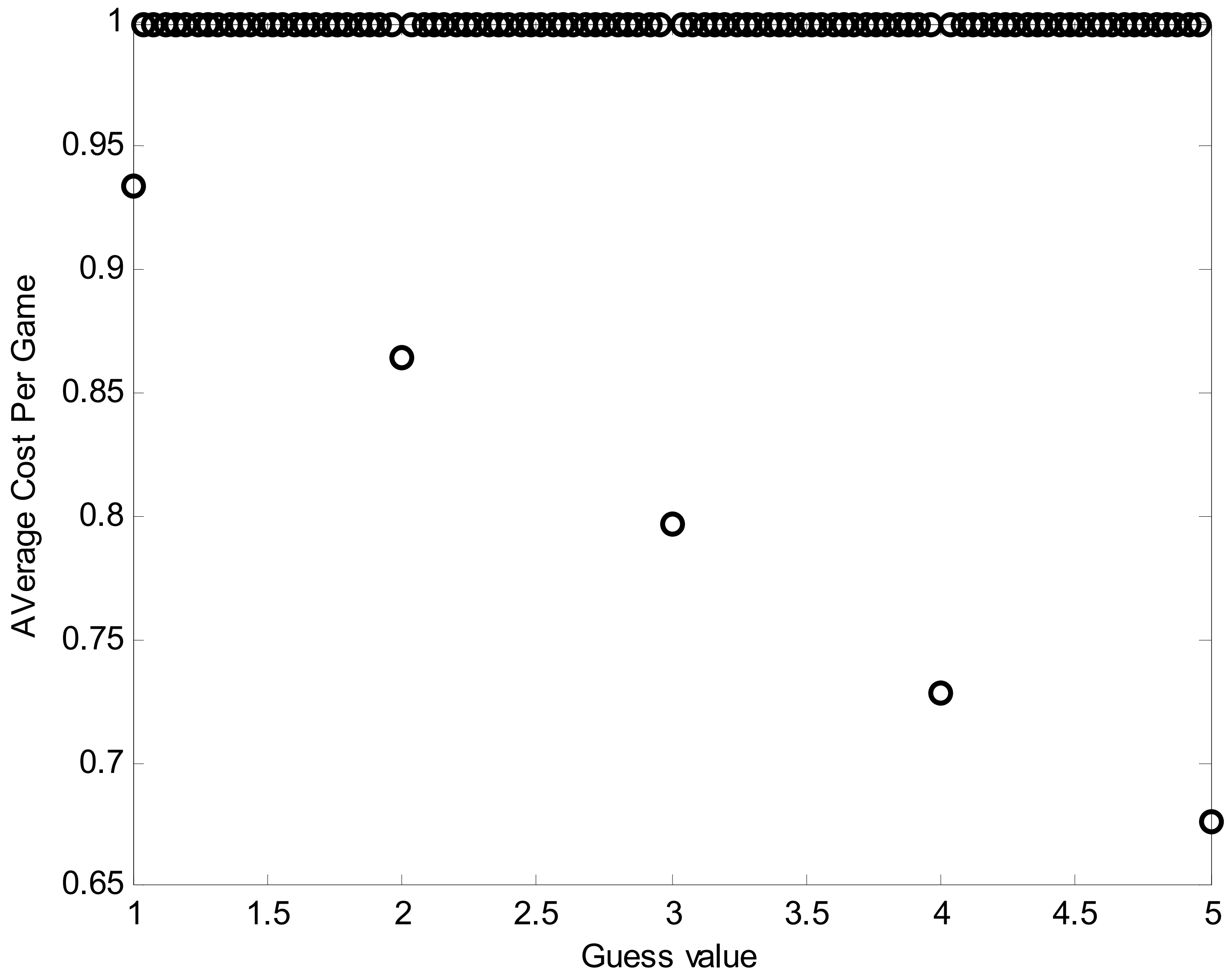
```
>> GuessingGame_4_1_1
X =
     3     5     1     2     5
g =
     1
cost =
     4
averageCostPerGame =
    0.8000
```

```
close all; clear all;

n = 10; % number of time to play this game

D = [1 2 2 3 3 3 4 4 4 4 5 5 5 5 5];
X = D(randi(length(D),1,n));

if n <= 10
    X
end

g = 3.3
cost = sum(X ~= g)

if n > 1
averageCostPerGame = cost/n
end
```

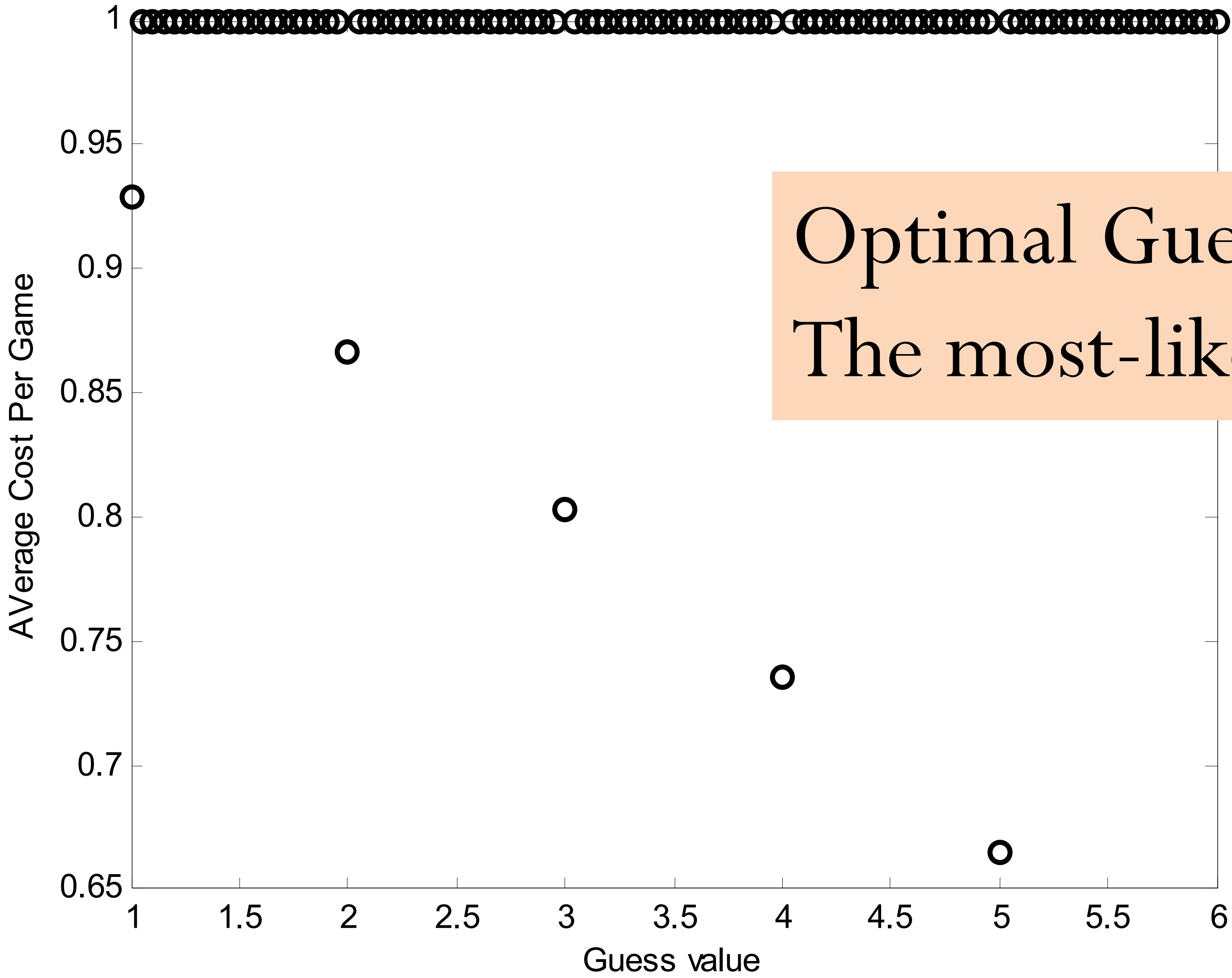
```
>> GuessingGame_4_1_1
X =
     5     3     2     4     1
g =
    3.3000
cost =
     5
averageCostPerGame =
     1
```

```
close all; clear all;

n = 1e4; % number of time to play this game

D = [1 2 2 3 3 3 4 4 4 4 5 5 5 5 5];
X = D(randi(length(D),1,n));

if n <= 10
    X
end

g = ?
cost = sum(X ~= g)

if n > 1
averageCostPerGame = cost/n
end
```

Guessing Game 1



Guessing Game 1



Optimal Guess:
The most-likely value

Guessing Game 2

- There are 15 cards.
  - Each have a number on it.
  - Here are the 15 cards:

1 2 2 3 3 3 4 4 4 4 5 5 5 5 5

- One card is randomly selected from the 15 cards.
- You need to guess the number X on the card.
- Suppose your guess value is g. The amount that you have to pay for incorrect guess is

$$(X - g)^2.$$

- The game is to be repeated $n = 10{,}000$ times.
- What should be your guess value?

```matlab
close all; clear all;

n = 3; % number of time to play this game

D = [1 2 2 3 3 3 4 4 4 4 5 5 5 5 5];
X = D(randi(length(D),1,n));

if n <= 10
    X
end

g = 5
cost = sum((X-g).^2)

if n > 1
averageCostPerGame = cost/n
end
```

```
>> GuessingGame_4_2_1
X =
     2     5     3
g =
     5
cost =
    13
averageCostPerGame =
    4.3333
```

```matlab
close all; clear all;

n = 1e4; % number of time to play this game

D = [1 2 2 3 3 3 4 4 4 4 5 5 5 5 5];
X = D(randi(length(D),1,n));

if n <= 10
    X
end

g = 5
cost = sum((X-g).^2)

if n > 1
averageCostPerGame = cost/n
end
```

```
>> GuessingGame_4_2_1
g =
     5
cost =
        33169
averageCostPerGame =
    3.3169
```

```
close all; clear all;

n = 1e4; % number of time to play this game

D = [1 2 2 3 3 3 4 4 4 4 5 5 5 5 5];
X = D(randi(length(D),1,n));

if n <= 10
    X
end

g = 4.5
cost = sum((X-g).^2)

if n > 1
averageCostPerGame = cost/n
end
```

```
>> GuessingGame_4_2_1
g =
    4.5000
cost =
       22464
averageCostPerGame =
    2.2464
```

```
close all; clear all;

n = 1e4; % number of time to play this game

D = [1 2 2 3 3 3 4 4 4 4 5 5 5 5 5];
X = D(randi(length(D),1,n));

if n <= 10
    X
end

g = 4.5
cost = sum((X-g).^2)

if n > 1
averageCostPerGame = cost/n
end
```

Guessing a value that is not one of the original numbers is OK (and can be quite good) for this game.

```
>> GuessingGame_4_2_1
g =
    4.5000
cost =
       22464
averageCostPerGame =
    2.2464
```

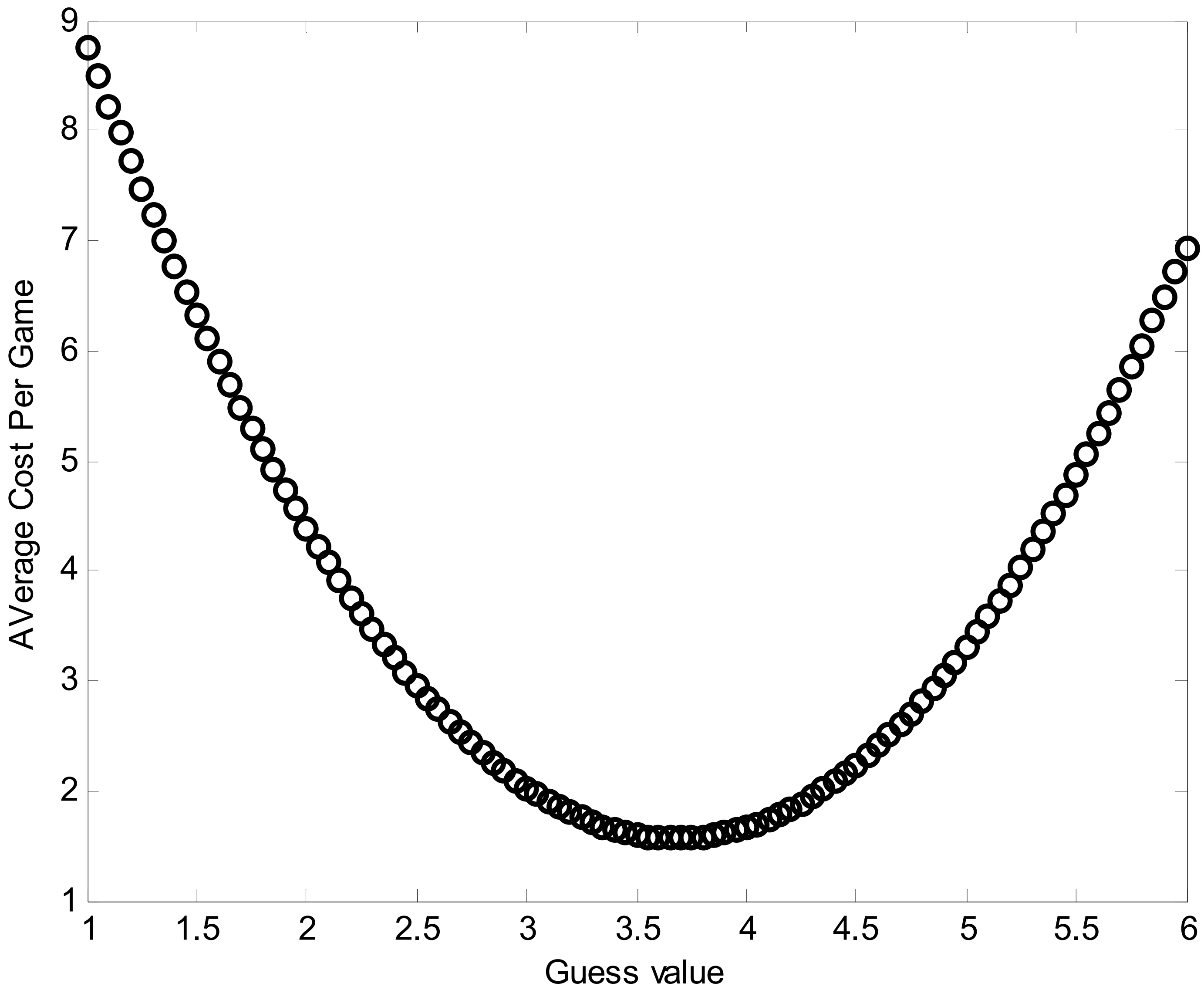
Guessing Game 2

- Suppose your guess value is g. The amount that you have to pay for incorrect guess is

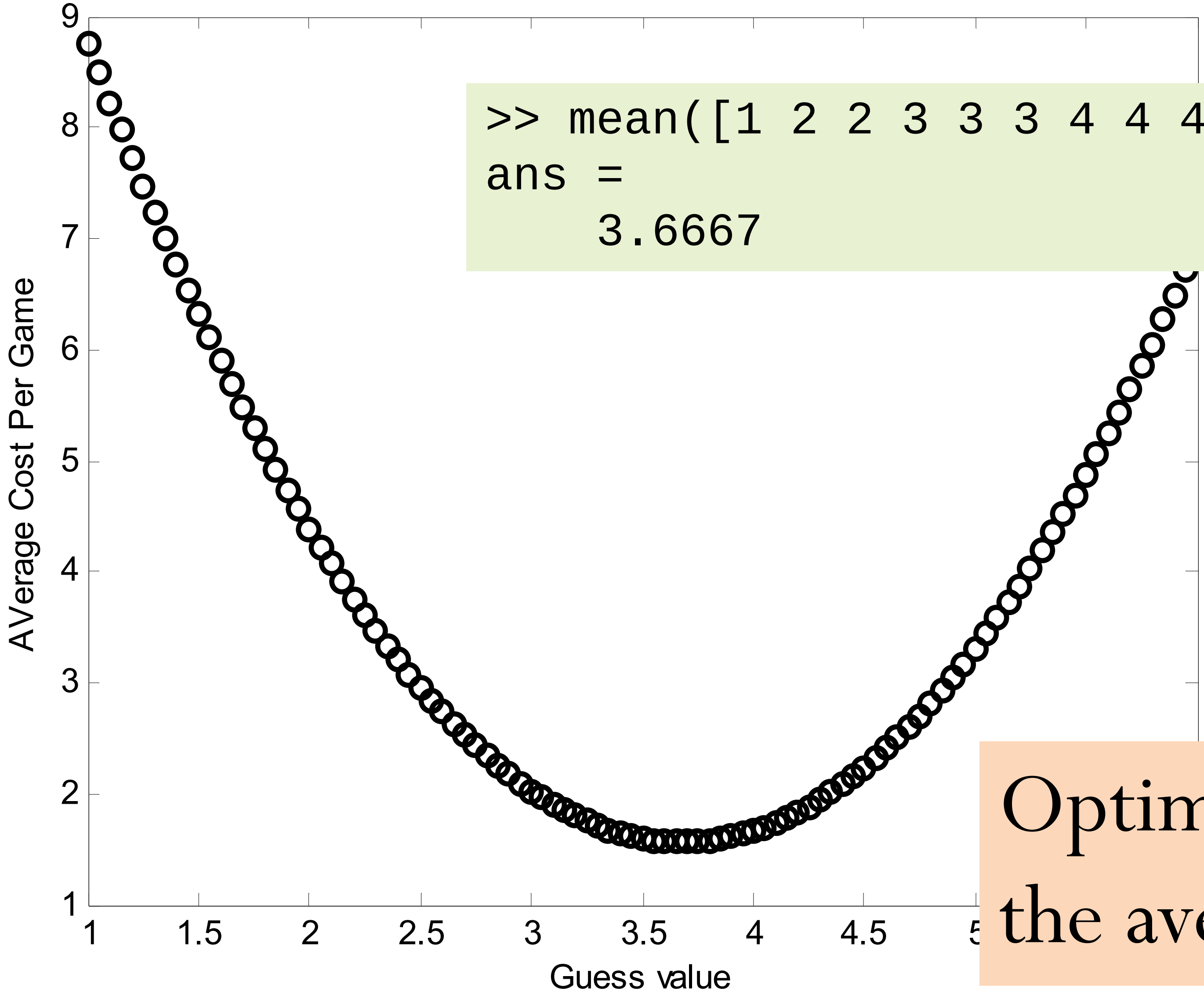
$$(X - g)^2.$$

- So, you want to minimize the square error.
  - Least-square.
  - Minimum Mean Square Error (MMSE) Estimator.

Guessing Game 2



Guessing Game 2

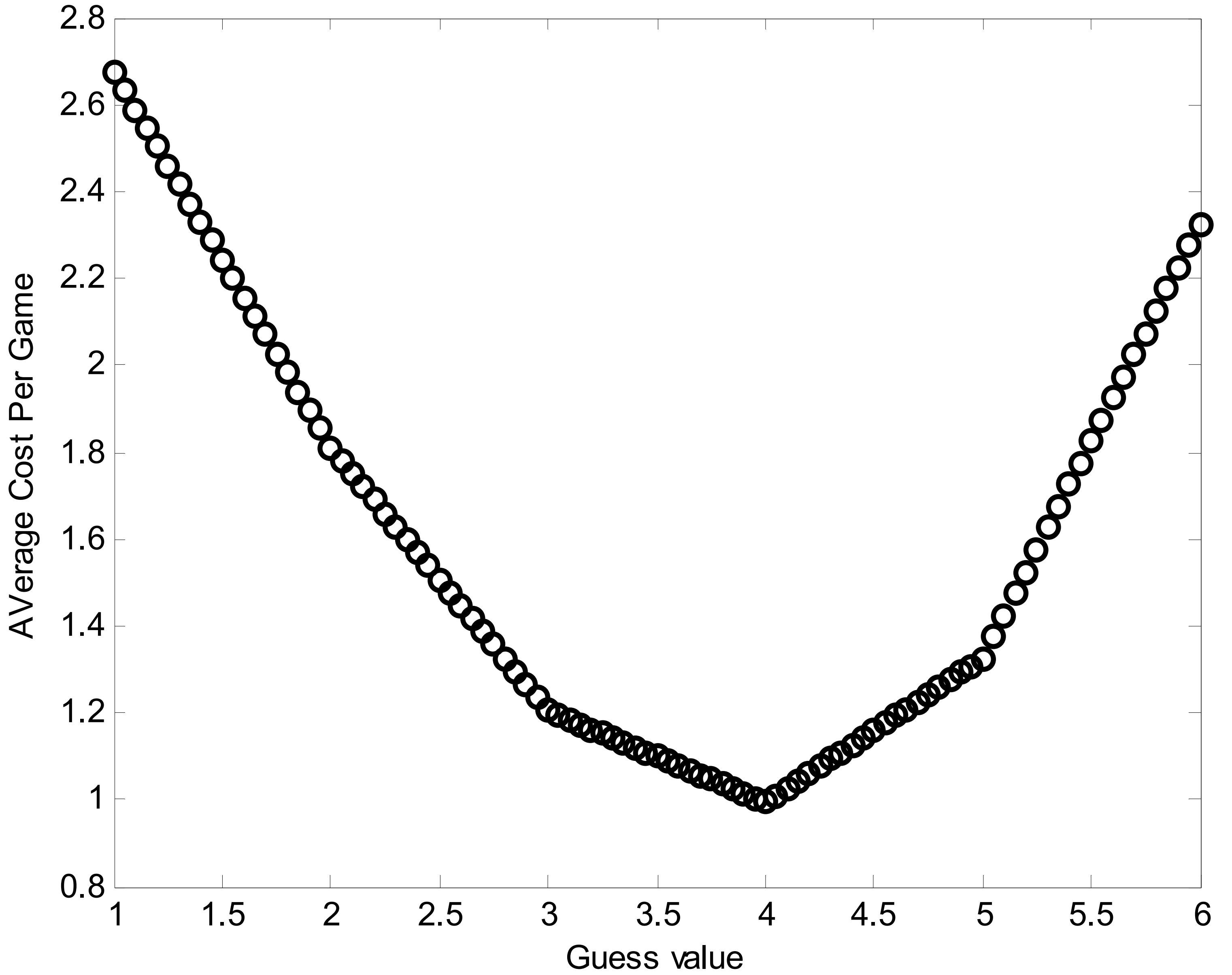



```
>> mean([1 2 2 3 3 3 4 4 4 4 5 5 5 5 5])
ans =
     3.6667
```

Optimal Guess: the average value

Guessing Game 3

- Suppose your guess value is g. The amount that you have to pay for incorrect guess is
$$|X - g|.$$
- So, you want to minimize the absolute error.
  - Least-square.
  - Minimum Mean Absolute Error (MMAE) Estimator.


AVerage Cost Per Game
Guess value
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6

Guessing Game 3

- There are 15 cards.
  - Each have a number on it.
  - Here are the 15 cards:

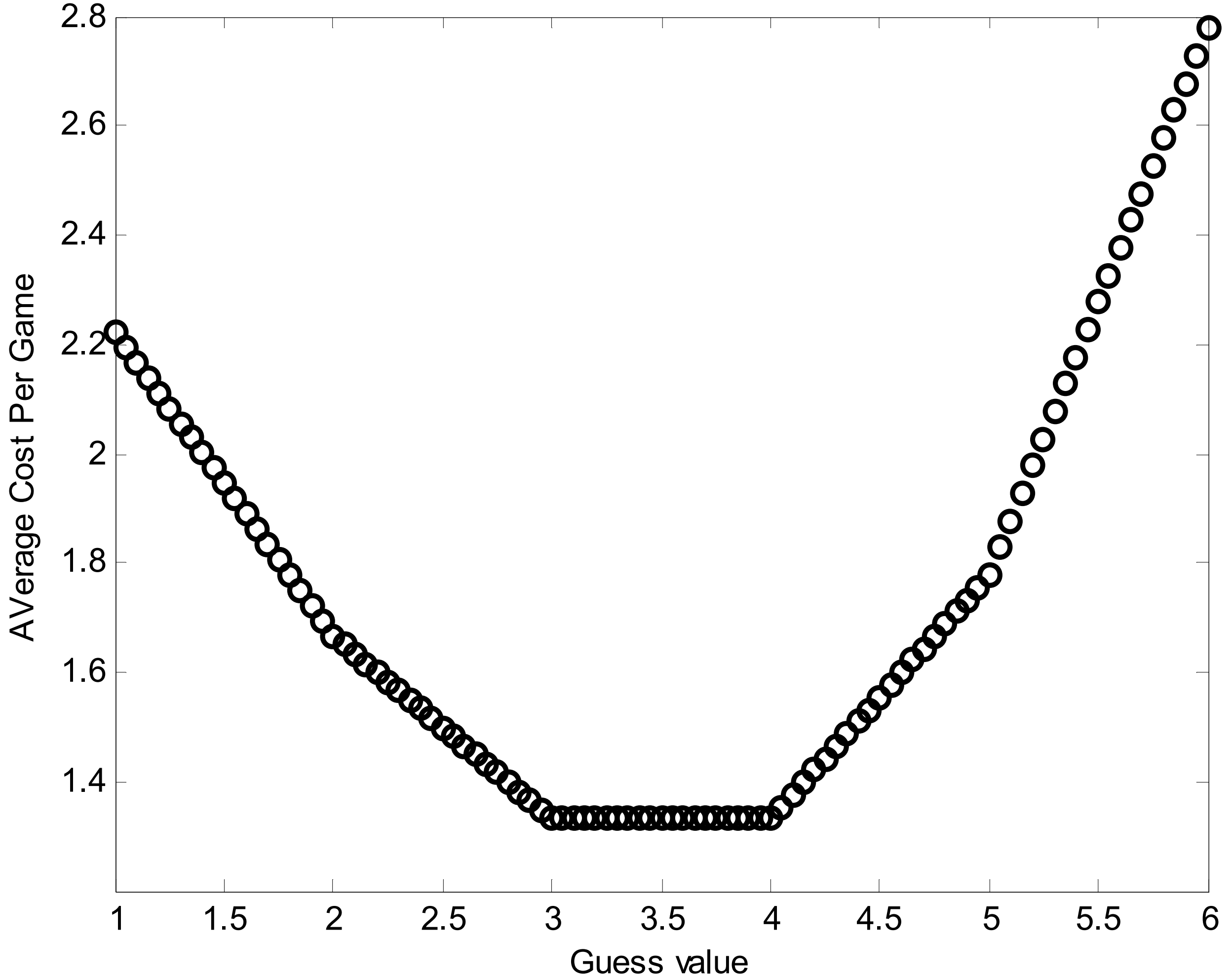
1 2 2 3 3 3 4 4 4 4 5 5 5 5 5

4 is the median of these numbers

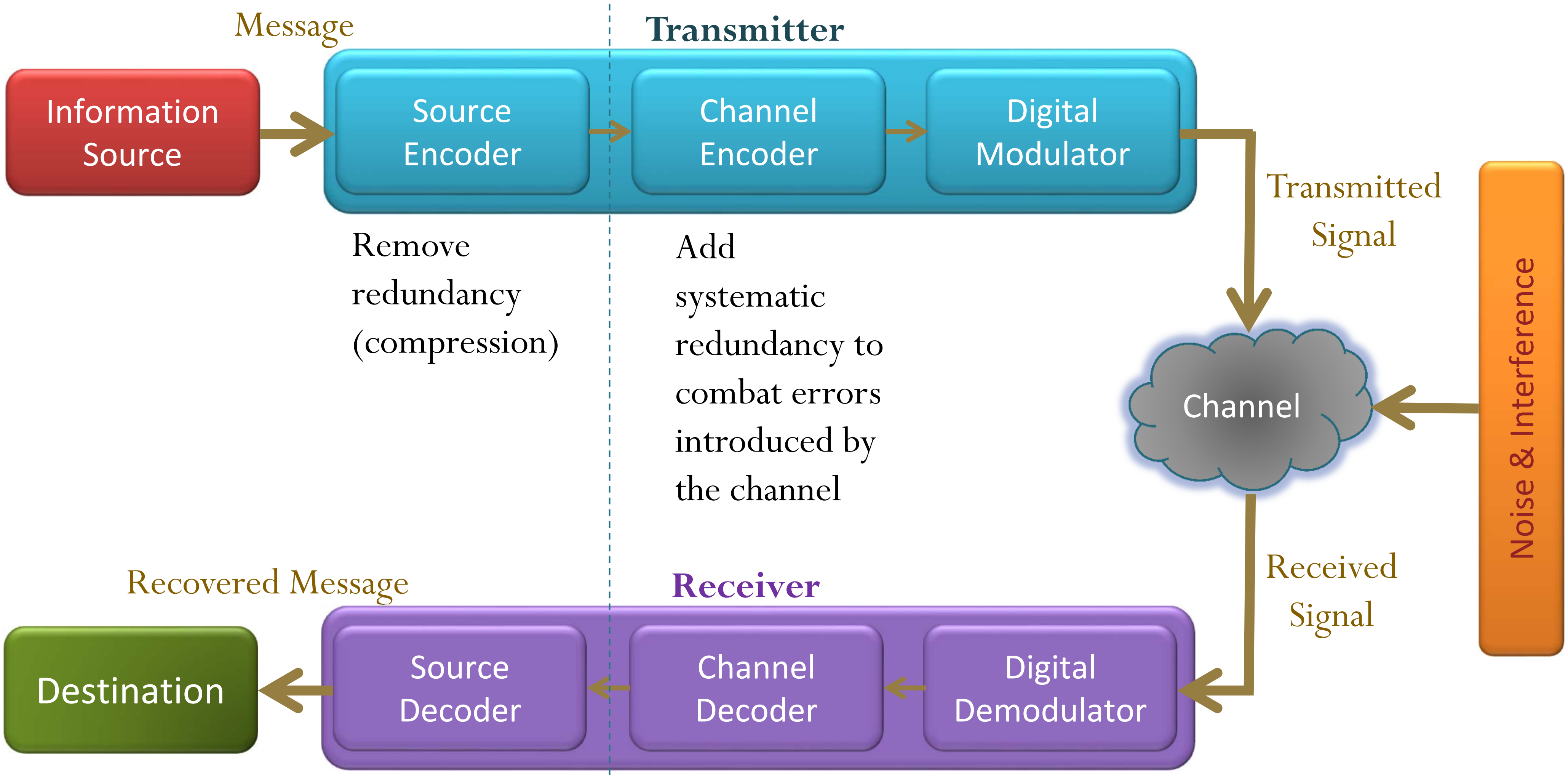
Guessing Game 3

- Suppose we have 18 cards:

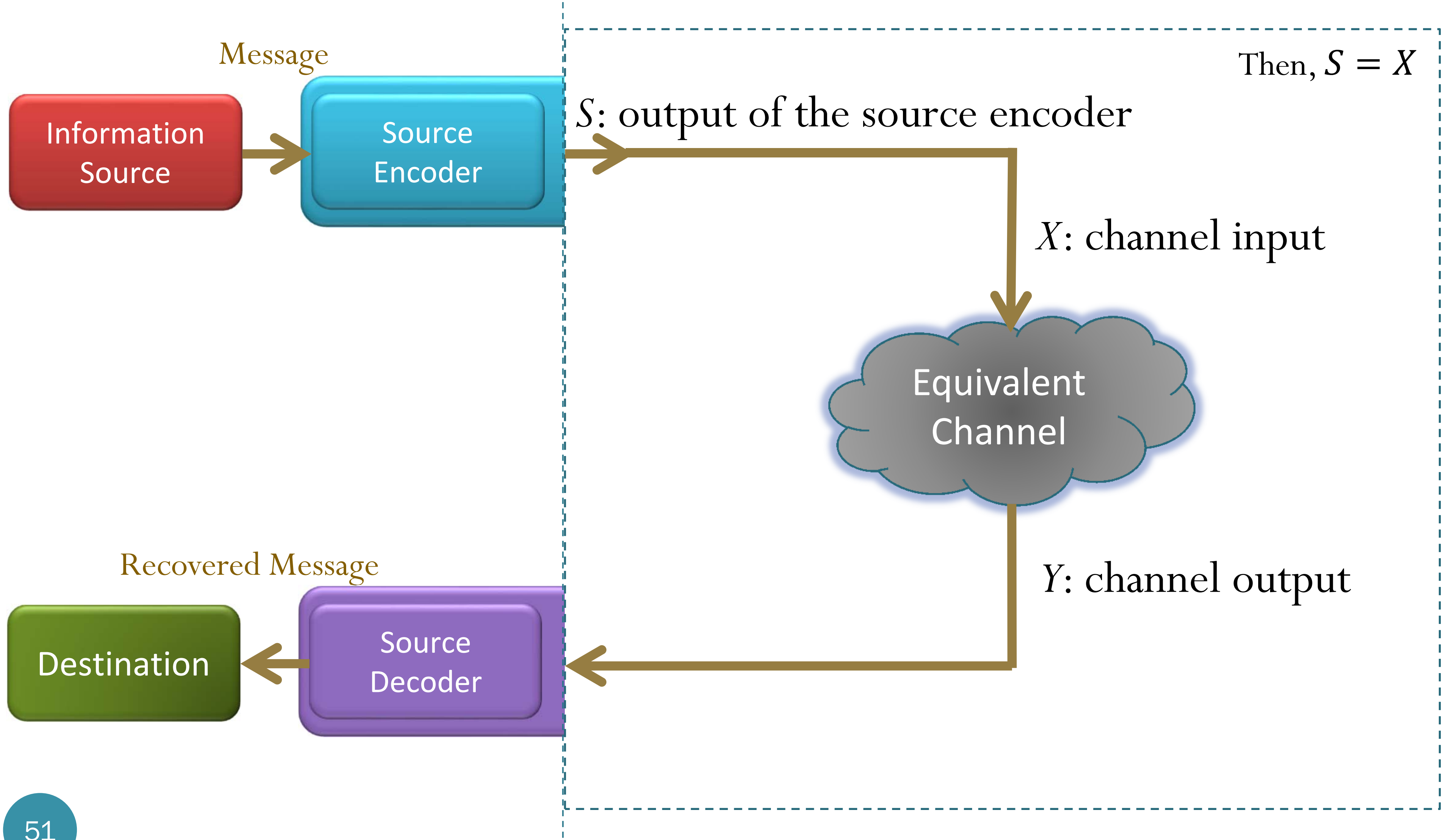
1 1 1 1 2 2 3 3 3 4 4 4 4 5 5 5 5 5



Elements of digital commu. sys.



System considered



The ASCII Coded Character Set

Bit Number					***6***	***0***	***0***	***0***	***0***	***1***	***1***	***1***	***1***
					5	***0***	***0***	***1***	***1***	***0***	***0***	***1***	***1***
					4	***0***	***1***	***0***	***1***	***0***	***1***	***0***	***1***
				Hex	***1st***	***0***	***1***	***2***	***3***	***4***	***5***	***6***	***7***
3	***2***	***1***	***0***	***2nd***									
						0	16	32	48	64	80	96	112
0	0	0	0	0		NUL	DLE	SP	0	@	P	`	p
0	0	0	1	1		SOH	DC1	!	1	A	Q	a	q
0	0	1	0	2		STX	DC2	“	2	B	R	b	r
0	0	1	1	3		ETX	DC3	#	3	C	S	c	s
0	1	0	0	4		EOT	DC4	$	4	D	T	d	t
0	1	0	1	5		ENQ	NAK	%	5	E	U	e	u
0	1	1	0	6		ACK	SYN	&	6	F	V	f	v
0	1	1	1	7		BEL	ETB	'	7	G	W	g	w
1	0	0	0	8		BS	CAN	(	8	H	X	h	x
1	0	0	1	9		HT	EM	)	9	I	Y	i	y
1	0	1	0	A		LF	SUB	*	:	J	Z	j	z
1	0	1	1	B		VT	ESC	+	;	K	[	k	{
1	1	0	0	C		FF	FS	,	<	L	\	l	|
1	1	0	1	D		CR	GS	-	=	M	]	m	}
1	1	1	0	E		SO	RS	.	>	N	^	n	~
1	1	1	1	F		SI	US	/	?	O	—	o	DEL

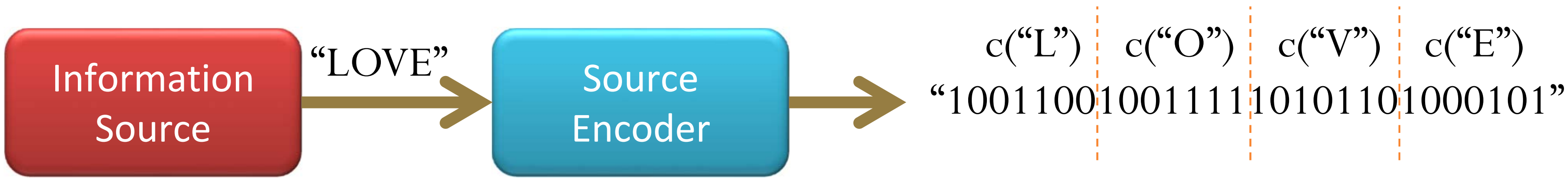
[The ARRL Handbook for Radio Communications 2013]

Example: ASCII Encoder

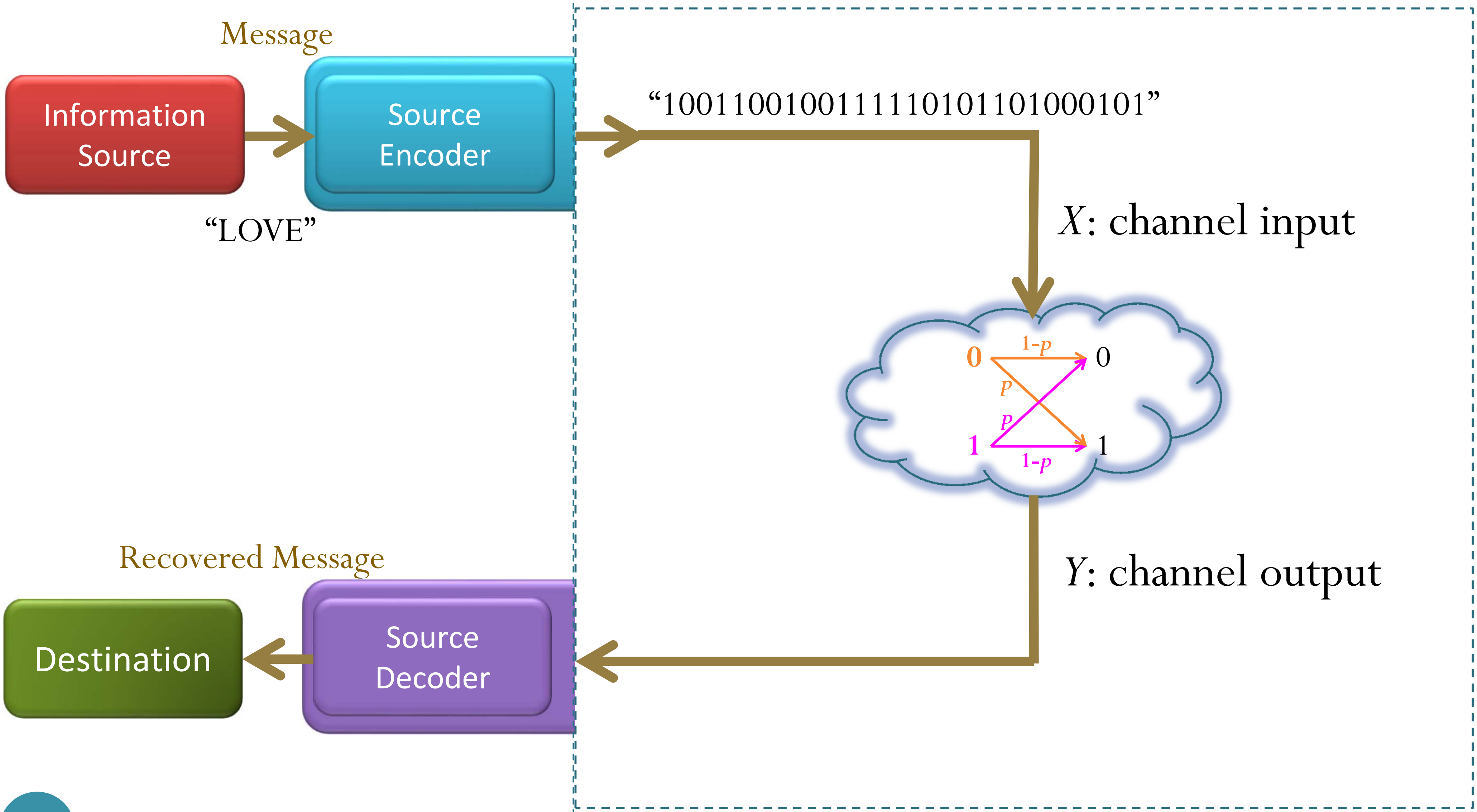
Character	Codeword
⋮	
E	1000101
⋮	
L	1001100
⋮	
O	1001111
⋮	
V	1010110
⋮	

MATLAB:

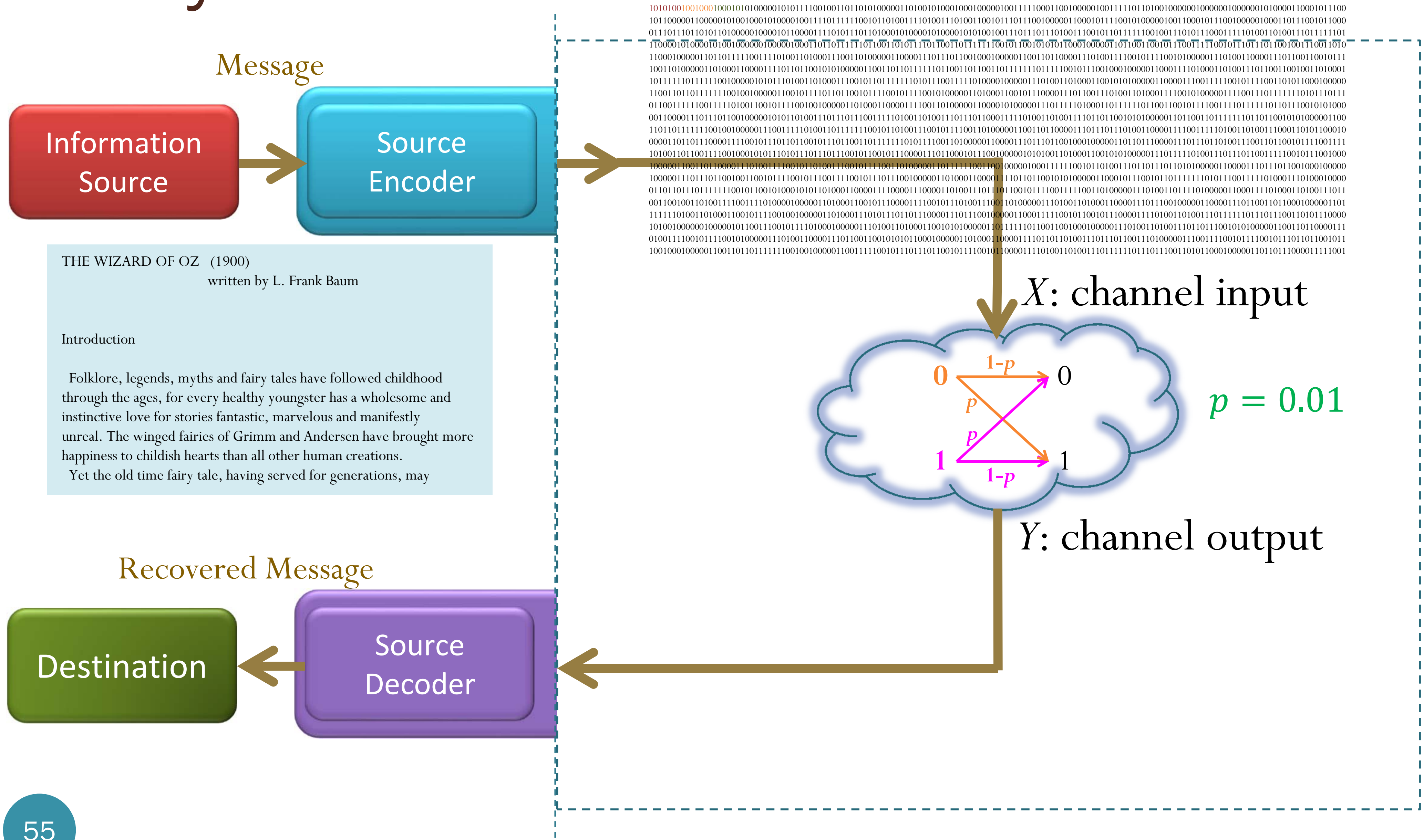
```
>> M = 'LOVE';
>> X = dec2bin(M,7);
>> X = reshape(X',1,numel(X))
X =
1001100100111110101101000101
```



Example: ASCII Encoder and BSC



System considered



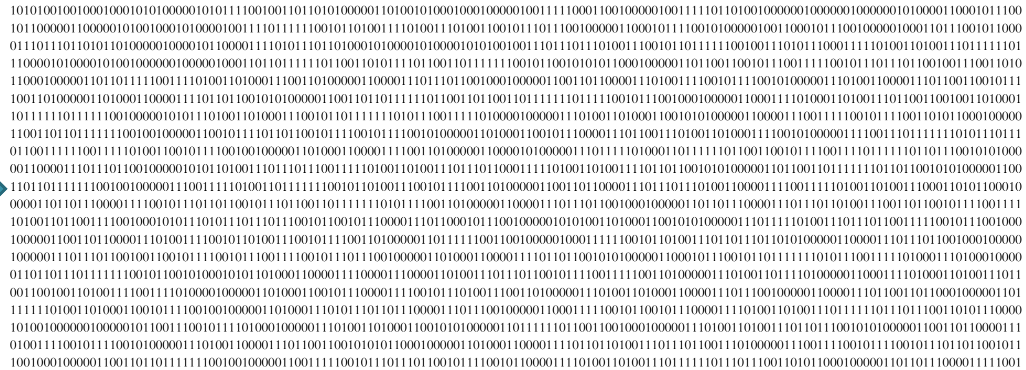
[ErrorProbabilityoverBSC.m]

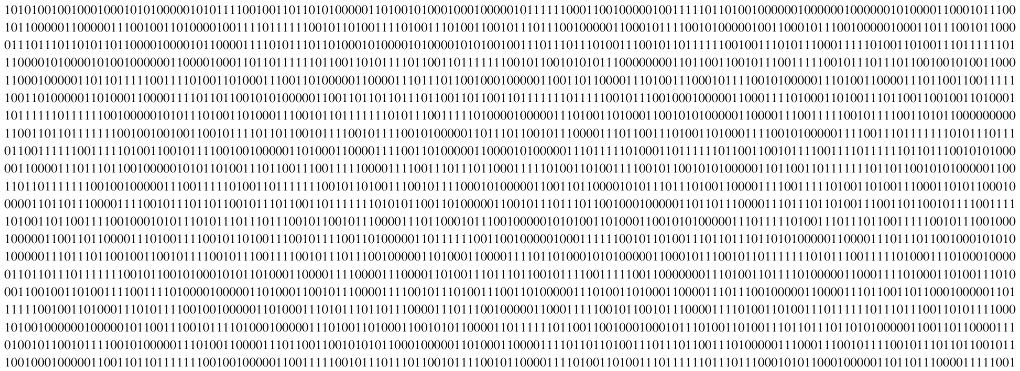
Results

THE WIZARD OF OZ (1900)
written by L. Frank Baum

Introduction

Folklore, legends, myths and fairy tales have followed childhood through the ages, for every healthy youngster has a wholesome and instinctive love for stories fantastic, marvelous and manifestly unreal. The winged fairies of Grimm and Andersen have brought more happiness to childish hearts than all other human creations.
Yet the old time fairy tale, having served for generations, may





THE WIZARD _F OZ (19009 written by L. Frank0Baum

Introduction

0Folklore. legendS myths and faiby talgs have fmllowed childhood through the ages, for$every nealthy youngster has a wholesome and ilspynctire love for storieq fa.tastic, marvelou3 end manifestly unreal. The winged fairies of Grimm and*Andersen havE brought more happiness to chihdish hearts than all odhur human creations/
Yet the0old"timm fai2y tale, having qerved for generationq, may

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Yet the0old"timm fai2y tale, having qerved for generationq, may

- The whole book which is saved in the file “OZ.txt” has 207760 characters (symbols).
- The ASCII encoded string has $207760 \times 7 = 1454320$ bits.
- The channel corrupts 14545 bits.
- This corresponds to 14108 erroneous characters.

Results

```
>> ErrorProbabilityoverBSC
biterror =
       14545
BER =
   0.010001237691842
theoretical_BER =
   0.010000000000000
characterErrror =
       14108
CER =
   0.067905275317674
theoretical_CER =
   0.067934652093010
```

$$\frac{14545}{1454320} \approx 0.01$$

$$\frac{14108}{207760} \approx 0.0679$$

- The file "OZ.txt" has 207760 characters (symbols).
- The ASCII encoded string has $207760 \times 7 = 1454320$ bits.
- The channel corrupts 14545 bits.
- This corresponds to 14108 erroneous characters.

Results

BSC's crossover probability

$$p = 0.01$$

$$\frac{14545}{1454320} \approx 0.01$$

$$\frac{14108}{207760} \approx 0.0679$$

$$\boxed{\text{CER} = 1 - (1 - p)^7}$$

- The file "OZ.txt" has 207760 characters (symbols).
- The ASCII encoded string has $207760 \times 7 = 1454320$ bits.
- The channel corrupts 14545 bits.
- This corresponds to 14108 erroneous characters (symbols).

A character (symbol) is successfully recovered if and only if none of its bits are corrupted.

Crossover probability and readability

When the first novel of the series, Harry Potter and the Philosopher's Stone (published in some countries as Harry Potter and the Sorcerer's Stone), opens, it is apparent that some significant event has taken place in the wizarding world--an event so very remarkable, even the Muggles notice signs of it. The full background to this event and to the person of Harry Potter is only revealed gradually through the series. After the introductory chapter, the book leaps forward to a time shortly before Harry Potter's eleventh birthday, and it is at this point that his magical background begins to be revealed.

Original

When the first novel of the series, Harry Pottez and the Philosopher's Stone (p5blished in some countries as Harry Potter cnd the Sorcerep's Stone), opens, it i3 apparent that soMe cignifacant event!haS taken0place in the wi~arding 7orld--ao event so`very!bemark!blu, even the Mufgles nodice signs"of it. The fuld background to this event and to the person of Harry P/tTer is only revealed gradually through th series. After the introfuctory chapter, the boo+ leaps forward to a time shortly before Harpy Potteb7s eleventh`birthday, and)t is at this poi~t that his -agikal bac{ground begins to be revealed.

$p = 0.01 \Rightarrow \text{CER} \approx 0.07$

Crossover probability and readability

Human may be able to correct some (or even all) of these errors.

When the first novel of the series, Harry Pottez and the Philosopher's Stone (p5blished in some countries as Harry Potter cnd the Sorcerep's Stone), opens, it i3 apparent that soMe cignifacant event!haS taken0place in the wi~arding 7orld--ao event so`very!bemark!blu, even the Mufgles nodice signs"of it. The fuld background to this event and to the person of Harry P/tTer is only revealed gradually through th series. After the introfuctory chapter, the boo+ leaps forward to a time shortly before Harpy Potteb7s eleventh`birthday, and)t is at this poi~t that his -agikal bac{ground begins to be revealed.

$p = 0.01 \Rightarrow \text{CER} \approx 0.07$

Crossover probability and readability

w(en th% birst .ovo,`of the serieq, Hcrry Potter(and the Phidosop`er's Suone (xub|ishe$(in some!Countries as @arry Potter and t`e Snr#erer's S|ong)- opens, it is apparent thatsoee smgfificant erent ha3 taieN place in the wizardino world-,an event!so very remarkable, even thE Euggl\ds notiae qigns of it. Tledfull back'tound to this event ane to the perron of Harry Popter is onl{ reveqned gsadeally thro}gH th%$serias. After the int2oducpory chcptur, the0jook deaps forward to"a!tmme shmrtly befosE Harry"Potter's eleventh jirthdiy cnd ht is a| thi3 po{nt tHat @is mAgiial background begijs to rm rerealed.

$p = 0.02 \Rightarrow \mathrm{CER} \approx 0.13$

Whethe first nOwgl nf the susi-Q-@Harr} PoutEr(and |he PhilosoxhEr's Ctonepuclyshed in som% coultries as Harrx @ tter and the S_rcermr7s Spone), opdns, id is apparent that {omg`signifikant evmnt ias taKen!Placa in tHe 7ijardIng world--an event so Very remaroqble, eve.!thE MugglaC fotice signc of"it. Uhe full backf2ound`to thas even| ant`0o the pEssoj of @arry Qotteb iw only revealed gradu!lly vhvoug` the rerier. Afte2 the IndRoductori chaptar,t`e book leats ForwaRf tc a 4imE shostl= before!Hcssy potter's u|Eveoth$firthdA{, and iT is ad this pomNt uhav `ir magica, back'bound cegins to bE 2evealed.

$p = 0.03 \Rightarrow \mathrm{CER} \approx 0.19$

Crossover probability and readability

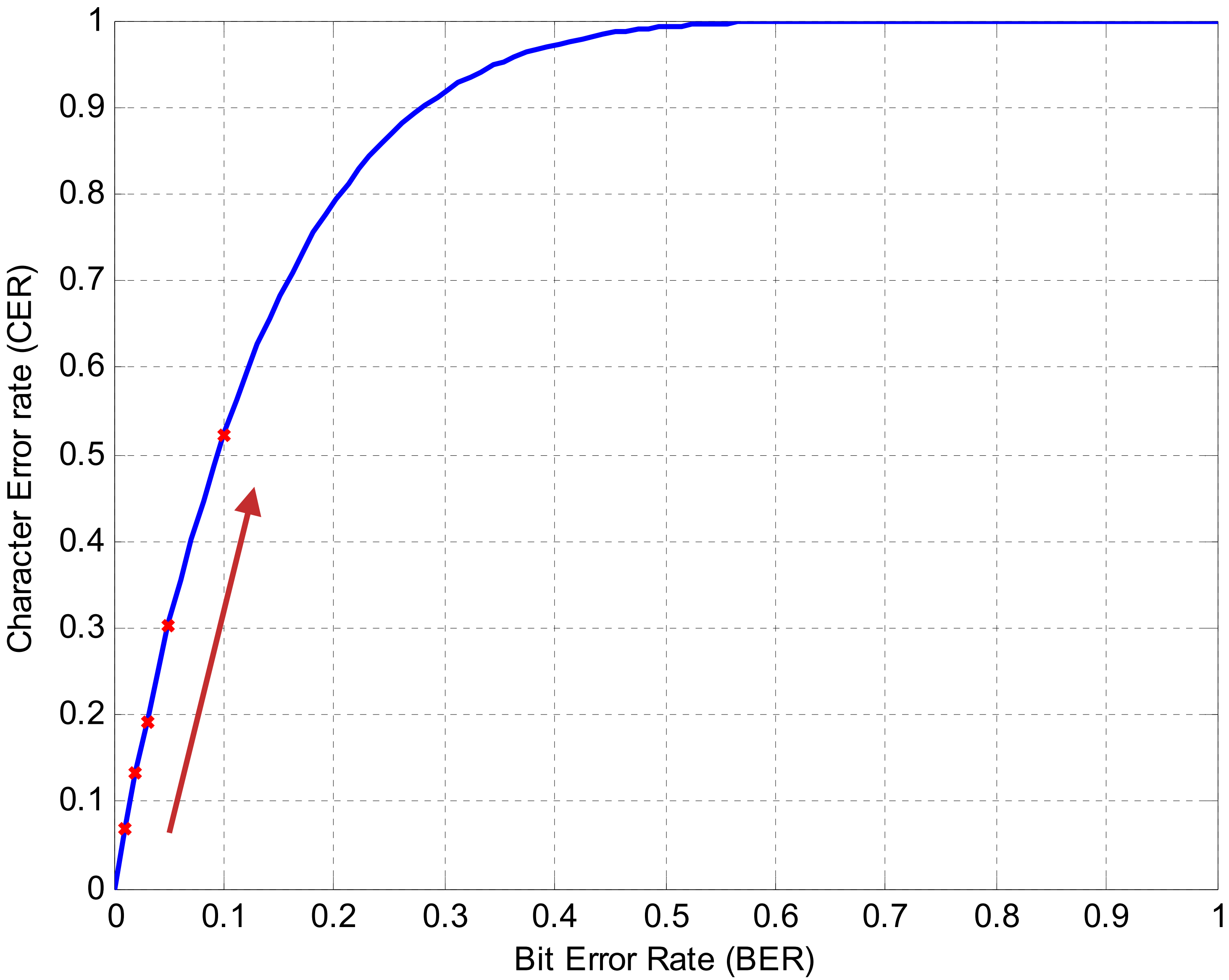
When phe fir   v okval ov"th% serie3, @`rry0Pntter efdxtxe Thil    soph%rs Stone0(p}blisjed!in {ooe c    un|pye{ agav0y Potter aj`(the sorcerer"s S4o|e)< opdns- mt"Is!apParEnt 4hat somu siwnidiga.v evant iAs take."plhge in(uhe w)zard)ng wo{|d--An event so very Rumar{ablel eteN0Dhe %ugcles$n   t)ce signs of$At. Tje!&ul|!backep/und Dk thkw`event ajt(to vhd per{On of8Ikxry P_Pter is oN,y rereAeud gredualli 4hroufh5ie qeriesn Af|ir the )~trofUctkry!ciapter,$tle r%ok lE`ps for   erd8to!a d)hg 3Hostly redobd HArry(Potter/r elaventI(birpl%ay,))nd(iD i3 1t tlis hohlt vhat iis$iagical bac+gropnd bedans to bg rEve!ied/

$p = 0.05 \Rightarrow \mathrm{CER} \approx 0.30$

Whef th% &i2sv nkvdl"On(txE"serm-s< HaRtY Qo|p%R$anlthe Phi$)qop8gb'r YtoNe (puclirhed in23/ee c   uNpr9es aZ Harby!PovDdZ qnd0THA!Uorojev's Qpof'),   pegsL iT is0aqazenP Tiet`{nle sau*!fICQ~t eve.t`xA# raken pOqb%%)D }Hm`wizprdYjv"wOrnd--a~%W%Jv s' tury 2maskABdd$`eden(tl| LuxGxec`nOtike c)gzq of kt Tiu!f5mm"cackG@    Ud(to"vhhQ a~aNd alt tn0vid veRckn of HaRvq$Xntter#isxohk{ regea,ed@&saduadLy u(2otGh"tau griEs."AfTex0T`g mntr   DUCt   ry kh `ter,$thd(fomN0j`apv ngrwarTt-0c t,me"1xortly bEemsL |ar2q Pnfter'3 aMen-n5i@Fipth$`q, aoh It i3d1t piac0pmhnP d*if Zas mafibin"je#k7poUndpb%dins tk`be qe6e!lgd.

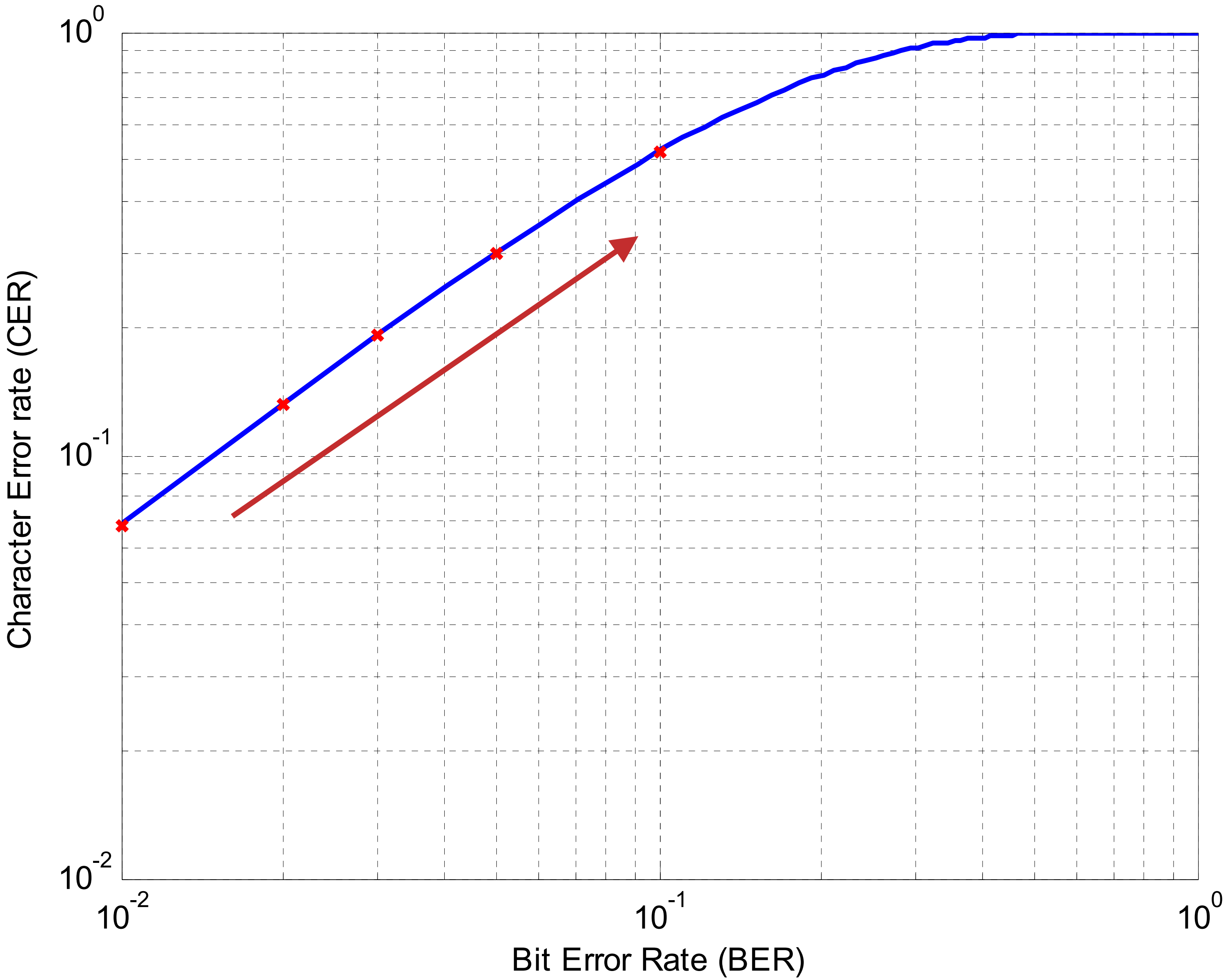
$p = 0.10 \Rightarrow \mathrm{CER} \approx 0.52$

BER vs. CER



Character Error rate (CER)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Bit Error Rate (BER)

BER vs. CER



10^0
10^{-1}
10^{-2}
Character Error rate (CER)
10^{-2}
10^{-1}
10^0
Bit Error Rate (BER)

Channel Encoder and Decoder